2022

AP® CollegeBoard

# AP® Calculus AB

## Scoring Guidelines

## Part A (AB or BC): Graphing calculator required

## Question 1 — 9 points

**General Scoring Notes**

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

From 5 A.M. to 10 A.M., the rate at which vehicles arrive at a certain toll plaza is given by $A(t) = 450\sqrt{\sin(0.62t)}$, where $t$ is the number of hours after 5 A.M. and $A(t)$ is measured in vehicles per hour. Traffic is flowing smoothly at 5 A.M. with no vehicles waiting in line.

| Model Solution | Scoring | |
|---|---|---|

**(a)** Write, but do not evaluate, an integral expression that gives the total number of vehicles that arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).

| | | |
|---|---|---|
| The total number of vehicles that arrive at the toll plaza from 6 A.M. to 10 A.M. is given by $\int_1^5 A(t)\,dt$. | Answer | **1 point** |

**Scoring notes:**

- The response must be a definite integral with correct lower and upper limits to earn this point.
- Because $|A(t)| = A(t)$ for $1 \le t \le 5$, a response of $\int_1^5 \left|450\sqrt{\sin(0.62t)}\right| dt$ or $\int_1^5 |A(t)|\,dt$ earns the point.
- A response missing $dt$ or using $dx$ is eligible to earn the point.
- A response with a copy error in the expression for $A(t)$ will earn the point only in the presence of $\int_1^5 A(t)\,dt$.

**Total for part (a) — 1 point**

**(b)** Find the average value of the rate, in vehicles per hour, at which vehicles arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).

| | | |
|---|---|---|
| $\text{Average} = \frac{1}{5-1}\int_1^5 A(t)\,dt = 375.536966$<br><br>The average rate at which vehicles arrive at the toll plaza from 6 A.M. to 10 A.M. is $375.537$ (or $375.536$) vehicles per hour. | Uses average value formula: $\frac{1}{b-a}\int_a^b A(t)\,dt$ | **1 point** |
| | Answer | **1 point** |

**Scoring notes:**

- The use of the average value formula, indicating that $a = 1$ and $b = 5$, can be presented in single or multiple steps to earn the first point. For example, the following response earns both points: $\int_1^5 A(t)\,dt = 1502.147865$, so the average value is $375.536966$.
- A response that presents a correct integral along with the correct average value, but provides incorrect or incomplete communication, earns 1 out of 2 points. For example, the following response earns 1 out of 2 points: $\int_1^5 A(t)\,dt = 1502.147865 = 375.536966$.
- The answer must be correct to three decimal places. For example, $\frac{1}{5-1}\int_1^5 A(t)\,dt = 375.536966 \approx 376$ earns only the first point.
- Degree mode: A response that presents answers obtained by using a calculator in degree mode does not earn the first point it would have otherwise earned. The response is generally eligible for all subsequent points (unless no answer is possible in degree mode or the question is made simpler by using degree mode). In degree mode, $\frac{1}{4}\int_1^5 A(t)\,dt = 79.416068$.
- Special case: $\frac{1}{5}\int_1^5 A(t)\,dt = 300.429573$ earns 1 out of 2 points.

**Total for part (b) 2 points**

**(c)** Is the rate at which vehicles arrive at the toll plaza at 6 A.M. ($t = 1$) increasing or decreasing? Give a reason for your answer.

| | | |
|---|---|---|
| $A'(1) = 148.947272$ | Considers $A'(1)$ | **1 point** |
| Because $A'(1) > 0$, the rate at which the vehicles arrive at the toll plaza is increasing. | Answer with reason | **1 point** |

**Scoring notes:**

- The response need not present the value of $A'(1)$. The second line of the model solution earns both points.
- An incorrect value assigned to $A'(1)$ earns the first point (but will not earn the second point).
- Without a reference to $t = 1$, the first point is earned by any of the following:
  - $148.947$ accurate to the number of decimals presented, with zero up to three decimal places (i.e., $149$, $148$, $148.9$, $148.95$, or $148.94$)
  - $A'(t) = 148.947$ by itself
- To be eligible for the second point, the first point must be earned.
- To earn the second point, there must be a reference to $t = 1$.
- Degree mode: $A'(1) = 23.404311$

**Total for part (c) 2 points**

**(d)** A line forms whenever $A(t) \geq 400$. The number of vehicles in line at time $t$, for $a \leq t \leq 4$, is given by $N(t) = \int_a^t (A(x) - 400)\,dx$, where $a$ is the time when a line first begins to form. To the nearest whole number, find the greatest number of vehicles in line at the toll plaza in the time interval $a \leq t \leq 4$. Justify your answer.

| | | |
|---|---|---|
| $N'(t) = A(t) - 400 = 0$<br>$\Rightarrow A(t) = 400 \Rightarrow t = 1.469372,\ t = 3.597713$ | Considers $N'(t) = 0$ | **1 point** |
| $a = 1.469372$<br>$b = 3.597713$ | $t = a$ and $t = b$ | **1 point** |

| $t$ | $N(t) = \int_a^t (A(x) - 400)\,dx$ |
|---|---|
| $a$ | 0 |
| $b$ | 71.254129 |
| 4 | 62.338346 |

The greatest number of vehicles in line is 71.

| | |
|---|---|
| Answer | **1 point** |
| Justification | **1 point** |

**Scoring notes:**

- It is not necessary to indicate that $A(t) = 400$ to earn the first point, although this statement alone would earn the first point.
- A response of "$A(t) \geq 400$ when $1.469372 \leq t \leq 3.597713$" will earn the first 2 points. A response of "$A(t) \geq 400$" along with the presence of exactly one of the two numbers above will earn the first point, but not the second. A response of "$A(t) \geq 400$" by itself will not earn either of the first 2 points.
- To earn the second point the values for $a$ and $b$ must be accurate to the number of decimals presented, with at least one and up to three decimal places. These may appear only in a candidates table, as limits of integration, or on a number line.
- A response with incorrect notation involving $t$ or $x$ is eligible to earn all 4 points.
- A response that does not earn the first point is still eligible for the remaining 3 points.
- To earn the third point, a response must present the greatest number of vehicles. This point is earned for answers of either $71$ or $71.254{*}{*}{*}$ only.
- A correct justification earns the fourth point, even if the third point is not earned because of a decimal presentation error.
- When using a Candidates Test, the response must include the values for $N(a)$, $N(b)$, and $N(4)$ to earn the fourth point. These values must be correct to the number of decimals presented, with up to three decimal places. (Correctly rounded integer values are acceptable.)
- Alternate solution for the third and fourth points:

  For $a \leq t \leq b$, $A(t) \geq 400$. For $b \leq t \leq 4$, $A(t) \leq 400$.

  Thus, $N(t) = \int_a^t (A(x) - 400)\, dx$ is greatest at $t = b$.

  $N(b) = 71.254129$, and the greatest number of vehicles in line is $71$.
- Degree mode: The response is only eligible to earn the first point because in degree mode $A(t) < 400$.

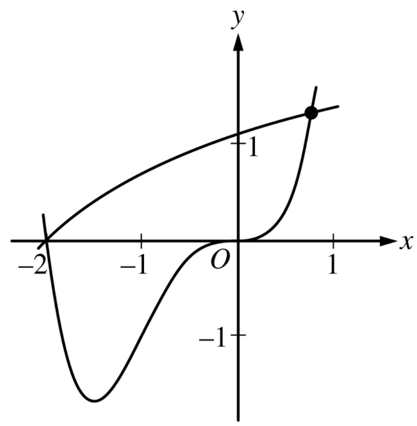
**Total for part (d) 4 points**

**Total for question 1 9 points**

## Part A (AB): Graphing calculator required
## Question 2 — 9 points

**General Scoring Notes**
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.




Let $f$ and $g$ be the functions defined by $f(x) = \ln(x+3)$ and $g(x) = x^4 + 2x^3$. The graphs of $f$ and $g$, shown in the figure above, intersect at $x = -2$ and $x = B$, where $B > 0$.

| Model Solution | Scoring | |
|---|---|---|
| **(a)** Find the area of the region enclosed by the graphs of $f$ and $g$. | | |
| $\ln(x+3) = x^4 + 2x^3 \Rightarrow x = -2,\ x = B = 0.781975$ | Integrand | **1 point** |
| $\int_{-2}^{B} (f(x) - g(x))\, dx = 3.603548$ | Limits of integration | **1 point** |
| The area of the region is $3.604$ (or $3.603$). | Answer | **1 point** |

**Scoring notes:**

- Other forms of the integrand in a definite integral, e.g., $|f(x) - g(x)|$, $|g(x) - f(x)|$, or $g(x) - f(x)$, earn the first point.
- To earn the second point, the response must have a lower limit of $-2$ and an upper limit expressed as either the letter $B$ with no value attached, or a number that is correct to the number of digits presented, with at least one and up to three decimal places.
    - Case 1: If the response did not earn the second point because of an incorrect value of $B$, $0 < B < 1$, but used a lower limit of $-2$, the response earns the third point only for a consistent answer.
    - Case 2: If the response did not earn the second point because the lower limit used was $x = 0$, but the response used a correct upper limit of $B$, the response earns the third point for a consistent answer of $0.708$ (or $0.707$).
    - Case 3: If a response uses any other incorrect limits it does not earn the second or third points.
- A response containing the integrand $g(x) - f(x)$ must interpret the value of the resulting integral correctly to earn the third point. For example, the following response earns all 3 points: $\int_{-2}^{B}(g(x) - f(x))\,dx = -3.604$ so the area is $3.604$. However, the response "Area $= \int_{-2}^{B}(g(x) - f(x))\,dx = 3.604$" presents an untrue statement and earns the first and second points but not the third point.
- A response must earn the first point in order to be eligible for the third point. If the response has earned the second point, then only the correct answer will earn the third point.
- Instructions for scoring a response that presents an integrand of $\ln(x + 3) - x^4 + 2x^3$ and the correct answer are shown in the "Global Special Case" after part (d).

**Total for part (a) 3 points**

**(b)** For $-2 \le x \le B$, let $h(x)$ be the vertical distance between the graphs of $f$ and $g$. Is $h$ increasing or decreasing at $x = -0.5$? Give a reason for your answer.

| | | |
|---|---|---|
| $h(x) = f(x) - g(x)$<br>$h'(x) = f'(x) - g'(x)$<br>$h'(-0.5) = f'(-0.5) - g'(-0.5) = -0.6$ (or $-0.599$) | Considers $h'(-0.5)$<br>– OR –<br>$f'(x) - g'(x)$ | **1 point** |
| Since $h'(-0.5) < 0$, $h$ is decreasing at $x = -0.5$. | Answer with reason | **1 point** |

**Scoring notes:**

- The response need not present the value of $h'(-0.5)$. The last line earns both points. However, if a value is presented it must be correct for the digits reported up to three decimal places.
- A response that reports an incorrect value of $h'(-0.5)$ earns only the first point.
- A response that presents only $h'(x)$ does not earn either point.
- The only response that earns the second point for concluding "$h$ is increasing" is described in the "Global Special Case" provided after part (d).
- A response that compares the values of $f'(x)$ and $g'(x)$ at $x = -0.5$ earns the first point and is eligible for the second point. This comparison can be made symbolically or verbally; for example, the response "the rate of change of $f(x)$ is less than the rate of change of $g(x)$ at $x = -0.5$" earns the first point.

**Total for part (b) 2 points**

**(c)** The region enclosed by the graphs of $f$ and $g$ is the base of a solid. Cross sections of the solid taken perpendicular to the $x$-axis are squares. Find the volume of the solid.

| | | |
|---|---|---|
| $\int_{-2}^{B} (f(x) - g(x))^2\, dx = 5.340102$ | Integrand | **1 point** |
| The volume of the solid is $5.340$. | Answer | **1 point** |

**Scoring notes:**

- The first point is earned for an integrand of $k(f(x) - g(x))^2$ or its equivalent with $k \neq 0$ in any definite integral. If $k \neq 1$, then the response is not eligible for the second point.
- A response that does not earn the first point is ineligible to earn the second point, with the following exceptions:
  - A response which has a presentation error in the integrand (for example, mismatched or missing parentheses, misplaced exponent) does not earn the first point but would earn the second point for the correct answer. A response which has a presentation error in the integrand and which reports an incorrect answer earns no points.
  - A response that presents an integrand of $\left(\ln(x+3) - x^4 + 2x^3\right)^2$. Scoring instructions for this case are provided in the "Global Special Case" after part (d).
- A response that uses incorrect limits is only eligible for the second point, provided the limits are imported from part (a) in Case 1 or Case 2. In both of these situations, the second point is earned only for answers consistent with the imported limits.

**Total for part (c) 2 points**

**(d)** A vertical line in the $xy$-plane travels from left to right along the base of the solid described in part (c). The vertical line is moving at a constant rate of $7$ units per second. Find the rate of change of the area of the cross section above the vertical line with respect to time when the vertical line is at position $x = -0.5$.

| | | |
|---|---|---|
| The cross section has area $A(x) = (f(x) - g(x))^2$.<br><br>$\frac{d}{dt}[A(x)] = \frac{dA}{dx} \cdot \frac{dx}{dt}$ | $\frac{dA}{dx} \cdot \frac{dx}{dt}$ | **1 point** |
| $\left.\frac{d}{dt}[A(x)]\right\rvert_{x=-0.5} = A'(-0.5) \cdot 7 = -9.271842$<br><br>At $x = -0.5$, the area of the cross section above the line is changing at a rate of $-9.272$ (or $-9.271$) square units per second. | Answer | **1 point** |

**Scoring notes:**

- The first point may be earned by presenting $\frac{dA}{dx} \cdot \frac{dx}{dt}$, $A' \cdot \frac{dx}{dt}$, $A'(x) \cdot \frac{dx}{dt}$, $A'(x) \cdot x'$, $A'(-0.5) \cdot 7$, or $k \cdot 7$, where $k$ is a declared value of $A'(-0.5)$, or any equivalent expression, including $2(f(x) - g(x))(f'(x) - g'(x))\frac{dx}{dt}$.
- If a response defines $f(x) - g(x)$ as a function in parts (b) or (c) (for example, $h(x) = f(x) - g(x)$), then a correct expression for $\frac{dA}{dt}$ (for example, $2h\frac{dh}{dt}$) earns the first point.
- A response that imports a function $A(x)$ declared in part (c) is eligible for both points (the answer must be consistent with the imported function $A(x)$).
- A response that presents an incorrect function for $A(x)$ that is not imported from part (c) is eligible only for the first point.
- Except when $A(x)$ is imported from part (c), the second point is earned only for the correct answer.
- A response that does not earn the first point is ineligible to earn the second point except in the special case noted below.

**Total for part (d) 2 points**

**Total for question 2 9 points**

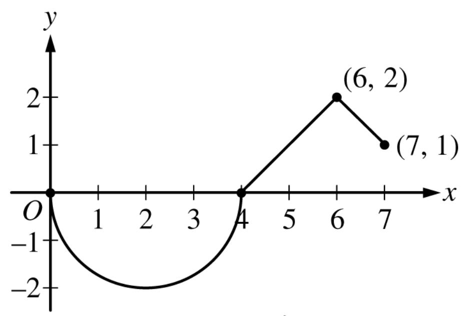
Global Special Case: A response may incorrectly simplify $f(x) - g(x)$ to $j(x) = \ln(x+3) - x^4 + 2x^3$ instead of $\ln(x+3) - x^4 - 2x^3$. Because this question is calculator active, a response with this incorrect simplification may nevertheless present correct answers.

- In any part of the question, a response that starts correctly by using $f(x) - g(x)$, then presents $j(x)$, is eligible for all points in that part.
- The first time a response implicitly presents $f(x) - g(x)$ as $j(x) = \ln(x+3) - x^4 + 2x^3$ (with no explicit connection) in any part of this question, the response loses a point. The response is then eligible for all remaining points for a correct or consistent answer.
- In part (a) the consistent answer using $j(x)$ is negative and will not earn the third point.
- In part (b) the consistent answer using $j(x)$ is that $j'(-0.5) = 2.4 > 0$, so $h$ is increasing at $x = -0.5$.
- In part (c) the consistent answer using $j(x)$ is $252.187$ (or $252.188$).
- In part (d) the consistent answer using $j(x)$ is $20.287$.

## Part B (AB or BC): Graphing calculator not allowed
## Question 3 — 9 points

**General Scoring Notes**
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.




Graph of $f'$

Let $f$ be a differentiable function with $f(4) = 3$. On the interval $0 \le x \le 7$, the graph of $f'$, the derivative of $f$, consists of a semicircle and two line segments, as shown in the figure above.

| | Model Solution | Scoring | |
|---|---|---|---|
| **(a)** | Find $f(0)$ and $f(5)$. | | |
| | $f(0) = f(4) + \int_4^0 f'(x)\,dx = 3 - \int_0^4 f'(x)\,dx = 3 + 2\pi$ <br><br> $f(5) = f(4) + \int_4^5 f'(x)\,dx = 3 + \frac{1}{2} = \frac{7}{2}$ | Area of either region <br> – OR – $\int_0^4 f'(x)\,dx$ <br> – OR – $\int_4^5 f'(x)\,dx$ | **1 point** |
| | | $f(0)$ | **1 point** |
| | | $f(5)$ | **1 point** |

**Scoring notes:**

- A response with answers of only $f(0) = \pm 2\pi$, or only $f(5) = \frac{1}{2}$, or both earns 1 of the 3 points.
- A response displaying $f(5) = \frac{7}{2}$ and a missing or incorrect value for $f(0)$ earns 2 of the 3 points.
- The second and third points can be earned in either order.
- Read unlabeled values from left to right and from top to bottom as $f(0)$ and $f(5)$. A single value must be labeled as either $f(0)$ or $f(5)$ in order to earn any points.

**Total for part (a) — 3 points**

**(b)** Find the $x$-coordinates of all points of inflection of the graph of $f$ for $0 < x < 7$. Justify your answer.

| | | |
|---|---|---|
| The graph of $f$ has a point of inflection at each of $x = 2$ and $x = 6$, because $f'(x)$ changes from decreasing to increasing at $x = 2$ and from increasing to decreasing at $x = 6$. | Answer | **1 point** |
| | Justification | **1 point** |

**Scoring notes:**

- A response that gives only one of $x = 2$ or $x = 6$, along with a correct justification, earns 1 of the 2 points.
- A response that claims that there is a point of inflection at any value other than $x = 2$ or $x = 6$ earns neither point.
- To earn the second point a response must use correct reasoning based on the graph of $f'$. Examples of correct reasoning include:
  - Correctly discussing the signs of the slopes of the graph of $f'$
  - Citing $x = 2$ and $x = 6$ as the locations of local extrema on the graph of $f'$
- Examples of reasoning not (sufficiently) connected to the graph of $f'$ include:
  - Reasoning based on sign changes in $f''$ unless the connection is made between the sign of $f''$ and the slopes of the graph of $f'$
  - Reasoning based only on the concavity of the graph of $f$
- The second point cannot be earned by use of vague or undefined terms such as "it" or "the function" or "the derivative."
- Responses that report inflection points as ordered pairs must report the points $(2, 3 + \pi)$ and $(6, 5)$ in order to earn the first point. If the $y$-coordinates are reported incorrectly, the response remains eligible for the second point.

**Total for part (b) 2 points**

**(c)** Let $g$ be the function defined by $g(x) = f(x) - x$. On what intervals, if any, is $g$ decreasing for $0 \le x \le 7$? Show the analysis that leads to your answer.

| | | |
|---|---|---|
| $g'(x) = f'(x) - 1$ | $g'(x) = f'(x) - 1$ | **1 point** |
| $f'(x) - 1 \le 0 \Rightarrow f'(x) \le 1$<br><br>The graph of $g$ is decreasing on the interval $0 \le x \le 5$ because $g'(x) \le 0$ on this interval. | Interval with reason | **1 point** |

**Scoring notes:**

- The first point can be earned for $f'(x) \le 1$ or the equivalent, in words or symbols.
- Endpoints do not need to be included in the interval to be eligible for the second point.

**Total for part (c) 2 points**

**(d)** For the function $g$ defined in part (c), find the absolute minimum value on the interval $0 \le x \le 7$. Justify your answer.

| | | |
|---|---|---|
| $g$ is continuous, $g'(x) < 0$ for $0 < x < 5$, and $g'(x) > 0$ for $5 < x < 7$.<br><br>Therefore, the absolute minimum occurs at $x = 5$, and $g(5) = f(5) - 5 = \frac{7}{2} - 5 = -\frac{3}{2}$ is the absolute minimum value of $g$. | Considers $g'(x) = 0$ | **1 point** |
| | Answer with justification | **1 point** |

**Scoring notes:**

- A justification that uses a local argument, such as "$g'$ changes from negative to positive (or $g$ changes from decreasing to increasing) at $x = 5$" must also state that $x = 5$ is the only critical point.
- If $g'(x) = 0$ (or equivalent) is not declared explicitly, a response that isolates $x = 5$ as the only critical number belonging to $(0, 7)$ earns the first point.
- A response that imports $g'(x) = f'(x)$ from part (c) is eligible for the first point but not the second.
  - In this case, consideration of $x = 4$ as the only critical number on $(0, 7)$ earns the first point.
- Solution using Candidates Test:

  $g'(x) = f'(x) - 1 = 0 \Rightarrow x = 5,\ x = 7$

  | $x$ | $g(x)$ |
  |---|---|
  | 0 | $3 + 2\pi$ |
  | 5 | $-\frac{3}{2}$ |
  | 7 | $-\frac{1}{2}$ |

  The absolute minimum value of $g$ on the interval $0 \le x \le 7$ is $-\frac{3}{2}$.

- When using a Candidates Test, a response may import an incorrect value of $f(0) = g(0) > -\frac{3}{2}$ from part (a). The second point can only be earned for an answer of $-\frac{3}{2}$.

**Total for part (d) 2 points**

**Total for question 3 9 points**

# Part B (AB or BC): Graphing calculator not allowed

## Question 4 — 9 points

**General Scoring Notes**

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

| $t$ (days) | 0 | 3 | 7 | 10 | 12 |
|---|---|---|---|---|---|
| $r'(t)$ (centimeters per day) | $-6.1$ | $-5.0$ | $-4.4$ | $-3.8$ | $-3.5$ |

An ice sculpture melts in such a way that it can be modeled as a cone that maintains a conical shape as it decreases in size. The radius of the base of the cone is given by a twice-differentiable function $r$, where $r(t)$ is measured in centimeters and $t$ is measured in days. The table above gives selected values of $r'(t)$, the rate of change of the radius, over the time interval $0 \le t \le 12$.

| Model Solution | Scoring | |
|---|---|---|

**(a)** Approximate $r''(8.5)$ using the average rate of change of $r'$ over the interval $7 \le t \le 10$. Show the computations that lead to your answer, and indicate units of measure.

| Model Solution | Scoring | |
|---|---|---|
| $r''(8.5) \approx \frac{r'(10) - r'(7)}{10 - 7} = \frac{-3.8 - (-4.4)}{10 - 7}$ | $r''(8.5)$ with supporting work | **1 point** |
| $= \frac{0.6}{3} = 0.2$ centimeter per day per day | Units | **1 point** |

**Scoring notes:**

- To earn the first point the supporting work must include at least a difference and a quotient.
- Simplification is not required to earn the first point. If the numerical value is simplified, it must be correct.
- The second point can be earned with an incorrect approximation for $r''(8.5)$ but cannot be earned without some value for $r''(8.5)$ presented.
- Units may be written in any equivalent form (such as $\text{cm}/\text{day}^2$).

**Total for part (a) — 2 points**

**(b)** Is there a time $t$, $0 \le t \le 3$, for which $r'(t) = -6$? Justify your answer.

| | | |
|---|---|---|
| $r(t)$ is twice-differentiable. $\Rightarrow$ $r'(t)$ is differentiable.<br>$\Rightarrow$ $r'(t)$ is continuous.<br><br>$r'(0) = -6.1 < -6 < -5.0 = r'(3)$<br>Therefore, by the Intermediate Value Theorem, there is a time $t$, $0 < t < 3$, such that $r'(t) = -6$. | $r'(0) < -6 < r'(3)$ | **1 point** |
| | Conclusion using Intermediate Value Theorem | **1 point** |

**Scoring notes:**

- To earn the first point, the response must establish that $-6$ is between $r'(0)$ and $r'(3)$ (or $-6.1$ and $-5$). This statement may be represented symbolically (with or without including one or both endpoints in an inequality) or verbally. A response of "$r'(t) = -6$ because $r'(0) = -6.1$ and $r'(3) = -5$" does not state that $-6$ is between $-6.1$ and $-5$. Thus this response does not earn the first point.
- To earn the second point:
  - The response must state that $r'(t)$ is continuous because $r'(t)$ is differentiable (or because $r(t)$ is twice differentiable).
  - The response must have earned the first point.
    - Exception: A response of "$r'(t) = -6$ because $r'(0) = -6.1$ and $r'(3) = -5$" does not earn the first point because of imprecise communication but may nonetheless earn the second point if all other criteria for the second point are met.
  - The response must conclude that there is a time $t$ such that $r'(t) = -6$. (A statement of "yes" would be sufficient.)
- To earn the second point, the response need not explicitly name the Intermediate Value Theorem, but if a theorem is named, it must be correct.

**Total for part (b) 2 points**

**(c)** Use a right Riemann sum with the four subintervals indicated in the table to approximate the value of $\int_0^{12} r'(t)\,dt$.

| | |
|---|---|
| $\int_0^{12} r'(t)\,dt \approx 3r'(3) + 4r'(7) + 3r'(10) + 2r'(12)$ | Form of right Riemann sum **1 point** |
| $= 3(-5.0) + 4(-4.4) + 3(-3.8) + 2(-3.5)$<br>$= -51$ | Answer **1 point** |

**Scoring notes:**

- To earn the first point, at least seven of the eight factors in the Riemann sum must be correct. If there is any error in the Riemann sum, the response does not earn the second point.
- A response of $3(-5.0) + 4(-4.4) + 3(-3.8) + 2(-3.5)$ earns both the first and second points, unless there is a subsequent error in simplification, in which case the response would earn only the first point.
- A response that presents the correct answer, with accompanying work that shows the four products in the Riemann sum (without explicitly showing all of the factors and/or the sum process) does not earn the first point but earns the second point. For example, $-15 + 4(-4.4) + 3(-3.8) + -7$ does not earn the first point but earns the second point. Similarly, $-15,\ -17.6,\ -11.4,\ -7 \rightarrow -51$ does not earn the first point but earns the second point.
- A response that presents the correct answer ( $-51$ ) with no supporting work earns no points.
- A response that provides a completely correct left Riemann sum and approximation $\int_0^{12} r'(t)\,dt$ (i.e., $3r'(0) + 4r'(3) + 3r'(7) + 2r'(10) = 3(-6.1) + 4(-5.0) + 3(-4.4) + 2(-3.8) = -59.1$) earns 1 of the 2 points. A response that has any error in a left Riemann sum or evaluation for $\int_0^{12} r'(t)\,dt$ earns no points.
- Units are not required or read in this part.

**Total for part (c) 2 points**

**(d)** The height of the cone decreases at a rate of $2$ centimeters per day. At time $t = 3$ days, the radius is $100$ centimeters and the height is $50$ centimeters. Find the rate of change of the volume of the cone with respect to time, in cubic centimeters per day, at time $t = 3$ days. (The volume $V$ of a cone with radius $r$ and height $h$ is $V = \frac{1}{3}\pi r^2 h$.)

| | | |
|---|---|---|
| $\frac{dV}{dt} = \frac{2}{3}\pi rh\frac{dr}{dt} + \frac{1}{3}\pi r^2\frac{dh}{dt}$ | Product rule | **1 point** |
| | Chain rule | **1 point** |
| $\left.\frac{dV}{dt}\right\|_{t=3} = \frac{2}{3}\pi(100)(50)(-5) + \frac{1}{3}\pi(100)^2(-2) = -\frac{70{,}000\pi}{3}$ <br><br> The rate of change of the volume of the sculpture at $t = 3$ is approximately $-\frac{70{,}000\pi}{3}$ cubic centimeters per day. | Answer | **1 point** |

**Scoring notes:**

- The first 2 points could be earned in either order.
- A response with a completely correct product rule, missing one or both of the correct differentials, earns the product rule point, but not the chain rule point. For example, $\frac{dV}{dt} = \frac{2}{3}\pi rh + \frac{1}{3}\pi r^2$ earns the first point, but not the second.
- A response that treats $r$ or $h$ (but not both) as a constant is eligible for the chain rule point but not the product rule point. For example, $\frac{dV}{dt} = \frac{2}{3}\pi rh\frac{dr}{dt}$ is correct if $h$ is constant, and thus earns the chain rule point.
- Note: Neither $\frac{dV}{dt} = \frac{2}{3}\pi r\frac{dh}{dt}$ nor $\frac{dV}{dt} = \frac{2}{3}\pi rh\frac{dr}{dt}\frac{dh}{dt}$ earns any points.
- A response that assumes a functional relationship between $r$ and $h$ (such as $r = 2h$), and uses this relationship to create a function for volume in terms of one variable, is eligible for at most the chain rule point. For example, $r = 2h \rightarrow V = \frac{1}{3}\pi(2h)^2 h = \frac{4}{3}\pi h^3 \rightarrow \frac{dV}{dt} = 4\pi h^2\frac{dh}{dt}$ earns only the chain rule point.
- A response that mishandles the constant $\frac{1}{3}\pi$ cannot earn the third point but is eligible for the first 2 points.
- The third point cannot be earned without both of the first 2 points.
- $\frac{dV}{dt} = \frac{2}{3}\pi(100)(50)(-5) + \frac{1}{3}\pi(100)^2(-2)$ earns all 3 points.
- Units are not required or read in this part.

**Total for part (d)** **3 points**

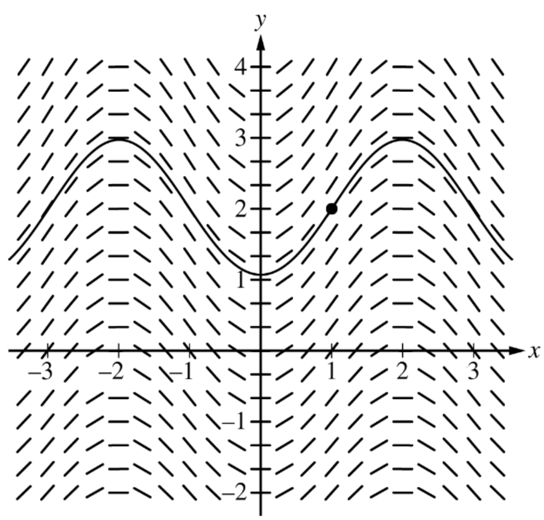
**Total for question 4** **9 points**

## Part B (AB): Graphing calculator not allowed
## Question 5 — 9 points

**General Scoring Notes**
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)\sqrt{y+7}$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = 2$. The function $f$ is defined for all real numbers.

| | Model Solution | Scoring | |
|---|---|---|---|
| **(a)** | A portion of the slope field for the differential equation is given below. Sketch the solution curve through the point $(1, 2)$. | | |
| |  | Solution curve | **1 point** |



**Scoring notes:**

- The solution curve must pass through the point $(1, 2)$, extend reasonably close to the left and right edges of the square and have no obvious conflicts with the given slope lines.
- Only portions of the solution curve within the given slope field are considered.
- The solution curve must indicate $f(x) > 0$ for all points on the curve.
- All local maximum/minimum points on the solution curve must occur at horizontal line segments in the slope field.

**Total for part (a)** — **1 point**

**(b)** Write an equation for the line tangent to the solution curve in part (a) at the point $(1, 2)$. Use the equation to approximate $f(0.8)$.

| | | |
|---|---|---|
| $\left.\dfrac{dy}{dx}\right\rvert_{(x,\, y)=(1,\, 2)} = \dfrac{1}{2} \cdot 3 \cdot \sin\left(\dfrac{\pi}{2}\right) = \dfrac{3}{2}$<br>An equation for the tangent line is $y = 2 + \frac{3}{2}(x - 1)$. | Tangent line equation | **1 point** |
| $f(0.8) \approx 2 + \frac{3}{2}(0.8 - 1) = 1.7$ | Approximation | **1 point** |

**Scoring notes:**

- The tangent line equation can be presented in any equivalent form.
- An incorrect tangent line equation with a slope of $\frac{3}{2}$ is eligible to earn the second point for a consistent answer.
- A response of only $2 + \frac{3}{2}(0.8 - 1)$ earns the second point but not the first.

**Total for part (b) 2 points**

**(c)** It is known that $f''(x) > 0$ for $-1 \le x \le 1$. Is the approximation found in part (b) an overestimate or an underestimate for $f(0.8)$? Give a reason for your answer.

| | | |
|---|---|---|
| Because $f''(x) > 0$, $f$ is concave up on $-1 \le x \le 1$, the tangent line lies below the graph of $y = f(x)$ at $x = 0.8$, and the approximation for $f(0.8)$ is an underestimate. | Answer with reason | **1 point** |

**Scoring notes:**

- The reason must include $f''(x) > 0$, $f'(x)$ is increasing, or $f(x)$ is concave up.

**Total for part (c) 1 point**

**(d)** Use separation of variables to find $y = f(x)$, the particular solution to the differential equation $\frac{dy}{dx} = \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)\sqrt{y+7}$ with the initial condition $f(1) = 2$.

| | | |
|---|---|---|
| $\int \frac{dy}{\sqrt{y+7}} = \int \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)dx$ | Separation of variables | **1 point** |
| $2\sqrt{y+7} = -\frac{1}{\pi}\cos\left(\frac{\pi}{2}x\right) + C$ | One correct antiderivative | **1 point** |
| | The other correct antiderivative | **1 point** |
| $f(1) = 2 \Rightarrow 2\sqrt{2+7} = -\frac{1}{\pi}\cos\left(\frac{\pi}{2}\cdot 1\right) + C$<br>$\Rightarrow 6 = -\frac{1}{\pi}\cos\left(\frac{\pi}{2}\right) + C \Rightarrow C = 6$<br>$\sqrt{y+7} = 3 - \frac{1}{2\pi}\cos\left(\frac{\pi}{2}x\right)$ | Constant of integration and uses initial condition | **1 point** |
| $y = \left(3 - \frac{1}{2\pi}\cos\left(\frac{\pi}{2}x\right)\right)^2 - 7$ | Solves for $y$ | **1 point** |

**Scoring notes:**

- A response with no separation of variables earns 0 out of 5 points.
- A response with no constant of integration can earn at most the first 3 points.
- A response is eligible for the fourth point only if it has earned the first point and at least 1 of the 2 antiderivative points.
    - Special case: The incorrect separation of $\sqrt{y+7}\ dy = \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)dx$ does not earn the first point, is only eligible for the antiderivative point for $-\frac{1}{\pi}\cos\left(\frac{\pi}{2}x\right)$, and is eligible for the fourth point.
- An eligible response earns the fourth point by correctly including the constant of integration in an equation and substituting $1$ for $x$ and $2$ for $y$.
- A response is eligible for the fifth point only if it has earned the first 4 points.

**Total for part (d) 5 points**

**Total for question 5 9 points**

## Part B (AB): Graphing calculator not allowed

## Question 6 — 9 points

**General Scoring Notes**

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Particle $P$ moves along the $x$-axis such that, for time $t > 0$, its position is given by $x_P(t) = 6 - 4e^{-t}$.

Particle $Q$ moves along the $y$-axis such that, for time $t > 0$, its velocity is given by $v_Q(t) = \frac{1}{t^2}$. At time $t = 1$, the position of particle $Q$ is $y_Q(1) = 2$.

| | Model Solution | Scoring | |
|---|---|---|---|
| **(a)** | Find $v_P(t)$, the velocity of particle $P$ at time $t$. | | |
| | $v_P(t) = x_P'(t) = 4e^{-t}$ | Answer | **1 point** |

**Scoring notes:**

- A response that equates $x_P(t)$ with $v_P(t)$ does not earn the point.
- An unlabeled response earns the point.

**Total for part (a)** **1 point**

**(b)** Find $a_Q(t)$, the acceleration of particle $Q$ at time $t$. Find all times $t$, for $t > 0$, when the speed of particle $Q$ is decreasing. Justify your answer.

| | | |
|---|---|---|
| $a_Q(t) = v_Q{}'(t) = \frac{-2}{t^3}$ | $a_Q(t)$ | **1 point** |
| For $t > 0$, $a_Q(t) < 0$ and $v_Q(t) > 0$. | Considers signs of $a_Q(t)$ and $v_Q(t)$ | **1 point** |
| Because the velocity and acceleration have opposite signs, the speed of particle $Q$ is decreasing for all $t > 0$. | Answer with justification | **1 point** |

**Scoring notes:**

- Earning the first point is not necessary for a response to be eligible to earn the second or third points; however, the response must present an expression for $a_Q(t)$ to be eligible for third point.
- A response earns the second point with either of the following statements: "$v_Q(t)$ and $a_Q(t)$ have opposite signs" or "$v_Q(t)$ and $a_Q(t)$ have the same sign." This statement, however, must be consistent with $v_Q(t)$ and the presented expression for $a_Q(t)$.
- A response must earn the second point to be eligible for the third point. The answer must be consistent with the presented justification. Furthermore, responses for which $a_Q(t) > 0$ for $t > 0$ must conclude that there is no time at which the speed of the particle is decreasing.
- A response that indicates $v_Q(t) < 0$ does not earn the third point, even if the answer and justification are consistent with a reported sign of $a_Q(t)$.

**Total for part (b) 3 points**

**(c)** Find $y_Q(t)$, the position of particle $Q$ at time $t$.

| | | |
|---|---|---|
| $y_Q(t) = y_Q(1) + \int_1^t \frac{1}{s^2}\,ds$ | Integral | **1 point** |
| | Uses initial condition | **1 point** |
| $= 2 - \left(\frac{1}{s}\Big\vert_1^t\right) = 2 - \frac{1}{t} + 1 = 3 - \frac{1}{t}$ | Answer | **1 point** |

**Scoring notes:**

- A response that presents $\int_1^t \frac{1}{t^2}\,dt$ (using the same variable as a limit and integrand function) does not earn the first point unless it is followed by an attempt at integration.
- A response that presents either $\int \frac{1}{t^2}\,dt$ or $-\frac{1}{t}$ (with no integral) earns the first point. If the response continues and presents $2 = -1 + C$, then the response earns the second point.
- A response that presents only $y_Q(t) = -\frac{1}{t} + 3$ will earn all 3 points. Note that the right side of this equation suffices to earn all points. A response of $y_Q(t) = -\frac{1}{t} + C$, where $C \neq 3$, (with no additional supporting work) earns only the first point.

**Total for part (c) 3 points**

**(d)** As $t \to \infty$, which particle will eventually be farther from the origin? Give a reason for your answer.

| | | |
|---|---|---|
| For particle $P$, $\lim_{t\to\infty}\left(6 - 4e^{-t}\right) = 6$.<br><br>For particle $Q$, $\lim_{t\to\infty}\left(3 - \frac{1}{t}\right) = 3$. | One correct limit | **1 point** |
| Because $6 > 3$, particle $P$ will eventually be farther from the origin. | Answer with reason | **1 point** |

**Scoring notes:**

- A response with an incorrect $y_Q(t)$ from part (c) is eligible for both points in part (d) provided $y_Q(t)$ is a non-constant function. The second point is earned for a consistent answer with reason, and limits correct for particle $P$ and the presented $y_Q(t)$.
- Responses that present statements such as "$6 - 4e^{-t}$ approaches $6$" or "$Q$ goes to $3$" earn the first point and are eligible for the second point.
- A response that treats $\infty$ as an input for $x_P(t)$ or $y_Q(t)$, such as "$6 - 4e^{-\infty}$" or "$3 - \frac{1}{\infty}$" is not eligible for the second point.

**Total for part (d) 2 points**

**Total for question 6 9 points**